# QUAID-I-AZAM UNIVERSITY

## ISLAMABAD

B.Sc. Annual Examinations--2013
(PART-I)

**Roll No: ______**

SUBJECT: **Mathematics-General** PAPER: **A**

Time Allowed: **3 Hours** **June 28, 2013** Max Marks: **100**

**Note:** **Attempt total FIVE questions selecting at least ONE and at the most TWO from each section. Use of only simple scientific calculator is allowed.**

### SECTION-I

**Q. No.1**

(a) Evaluate:

$$\lim_{x\to 0} \sin x / \sqrt[3]{x}$$ **(7)**

(b) Suppose:

$$f(x) = \begin{cases} c^2 x & \text{if } x < 1 \\ 3c\,x - 2 & \text{if } x \geq 1 \end{cases}$$

Determine all values of c such that f is continuous on R. **(7)**

(c) Find $g'(x)$, if:

$$g(x) = \sqrt{x^2+1}\ \ \sin\sqrt{x^2+1}$$ **(6)**

**Q. No.2**

(a) Find $y^{(n)}(0)$ if:

$$y = \ln(x + \sqrt{1+x^2})$$ **(7)**

(b) Use the 'Mean value Theorem' to show that:

$$1/6 < \sqrt{27} - 5 < 1/5$$ **(7)**

(c) Use "L'Hopital's Rule" to evaluate:`

$$\lim_{x\to 0} [x\cos x - \ln(1+x)]/x^2$$ **(6)**

**Q. No.3**

(a) A 20 meter long ladder placed against a wall. If the bottom of the ladder slips at the rate of 2m/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12 meter from the wall? **(7)**

(b) Find the intervals for which the curve

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7$$

(i). faces up (ii). faces down. **(6)**

(c) Prove that the radius of curvature on the curve

$$x^2y = a(x^2 + y^2)$$

at the point (2a, 2a) is 2a. **(7)**

### SECTION-II

**Q. No.4**

(a) Evaluate

$$\int (1 - \cos(\tfrac{t}{2}))^2\ \sin\left(\tfrac{t}{2}\right)\, dt$$ **(6)**

(b) Evaluate

$$\int dx/\sqrt{5x - 6 - x^2}$$ **(7)**

(c) Evaluate

$$\int_0^3 (2x-1)dx$$ by definition. **(7)**

*P.T.O*

**Q. No.5**

(a) Evaluate

$\int_0^{\pi/2} \ln(sinx)\ dx$ **(7)**

(b) Let $I_n = \int_0^{\infty} x^n\ e^{-x}\ dx$

where n is a positive integer. Prove that $I_n = n\ I_{n-1}$. Hence show that $I_n = n!$ **(7)**

(c) Use Simpson's rule to approximate:

$\int_0^1 \frac{1}{1+x^2}\ dx$ and $\pi$ with n = 4. **(6)**

**Q. No.6**

(a) Sketch the four leafed rose:

$\gamma = a \sin 2\theta$, where a > 0 **(7)**

(b) Find the area of the region included within the cardioids:

$\gamma = a(1 - \sin\theta)$. **(6)**

(c) Find the area of the surface of revolution about the x-axis the hypocycloid

$x = a\ cos^3\theta$, $y = a\ sin^3\theta$. **(7)**

## SECTION-III

**Q. No.7**

(a) Determine whether the series:

$\sum_1^{\infty} \ln n/(1+\ln n)$

is convergent or divergent. **(6)**

(b) Apply an appropriate test to examine the convergence or divergence of the series:

$\sum_0^{\infty} 2^n + n\ /(n+1)!$ **(7)**

(c) Determine the values of $x$ for which the series

$\sum_1^{\infty}(-1)^{n-1}\ \ x^n/\ n\ (n+1)$

i. converges absolutely

ii. converges conditionally

iii. diverges **(7)**

**Q. No.8**

(a) Find the Maclaurin's series for the function $f(x)$ given by

$f(x) = \ln(1 - x)$ **(7)**

(b) Find the value of "a" for which the function $f(x, y)$ given by

$$f(x,y) = \begin{cases} 3xy/\sqrt{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ a & \text{if } (x,y) = (0,0) \end{cases}$$ **(7)**

is continuous at the point (0,0).

(c) Find the points of inflection(s) for the curve:

$y^2 = x(x+1)^2$ **(6)**