QUAID-I-AZAM UNIVERSITY

ISLAMABAD

B.Sc. Annual Examinations--2013

(PART-I)

Roll No: ____________________

SUBJECT: **Mathematics A-Course** PAPER: **A** (Calculus & Analytic Geometry)

Time Allowed: **3 Hours** **June 11, 2013** Max Marks: **100**

Note: Attempt total FIVE questions selecting at least one and at the most two questions from each section. Use of only simple scientific calculator is allowed.

SECTION-I

Q. No.1

(a) Let $\delta > 0$ and $a \in R$. Show that $a - \delta < x < a + \delta$ if and only if $|x - a| < \delta$ **(6)**

(b) Evaluate $\lim\limits_{x \to 0} x^3 [1/x]$, [...] being the bracket function. **(7)**

(c) Solve the inequality: $2x/(x+2) \geq x/(x-2)$ and graph the solution set. **(7)**

Q. No.2

(a) Show that the function $y = |x - 1|$ is differentiable on $(-\infty, +1)$ and $(1, \infty)$ but has no derivative at $x = 1$. **(6)**

(b) If $y = (\arcsin x)^2$, prove that $y^{(n)} = 0$ when n is odd. **(7)**

(c) Let a function $y = f(x)$ is continuous on $[a, b]$ and differentiable in $]a, b[$ then there exists a point $c \in]a, b[$ such that $f(b) - f(a)/(b - a) = f'(c)$. **(7)**

Q. No.3

(a) Let $f(x) = \begin{cases} \sin 2x & \text{if } 0 < x \leq \pi/6 \\ ax + b & \text{if } \pi/6 < x \leq 1 \end{cases}$

Derive values of a and b so that $f(x)$ is continuous and differentiable at $x = \pi/6$. **(6)**

(b) Use the Newton-Raphson method to approximate upto four decimal places a root of $x^3 - 5x + 3 = 0$ with $x_0 = 0$. **(7)**

(c) Find Maclaurin's series for the function $f(x) = \ln(1+x)$. **(7)**

SECTION-II

Q. No.4

(a) Show that tangents at the ends of focal chord of a parabola intersect at right angle on the directrix. **(7)**

(b) Sketch the polar curve $r = a(1 - \sin\theta)$. (7)

(c) Show that in any conic, sum of the reciprocals of the segments of any focal chord is constant. (6)

Q. No.5

(a) Show that tangents to the cardiod $r = a(1 + \cos\theta)$ at the points $\theta = \pi/3$ and $\theta = 2\pi/6$ are respectively parallel and perpendicular to the initial line. **(7)**

(b) Show that the Pedal equation of the curve $x = a\cos^3\theta$, $y = \sin^3\theta$ is $r^2 = a^2 - 3p^2$ **(7)**

(c) Find equation of the straight line through the point A(5, -4, 4) and intersecting at right angle the straight line.

$$x/(-1) = (y - 1)/1 = z/(-2)$$ **(6)**

Q. No.6

(a) Find an equation of the plane passing through the line of intersection of the planes $2x - y + 2z = 0$ and $x + 2y - 2z - 3 = 0$ and at unit distance from the origin. **(7)**

(b) Find an equation of the cylinder with directrix C: $x^2/4 + y^2/9 = 1$ and having elements parallel to $n = [1, 1, 1]$. **(7)**

(c) Find an equation of the torus obtained by revolving about y-axis the circle in the xy-plane with centre at (a, 0, 0) and radius b, where $0<b<a$. **(6)**

SECTION-III

Q. No.7

(a) Show that height of an open cylinder of given surface S and the greatest volume is equal to the radius of its base. **(7)**

(b) Prove that for the cardiod
$r = a\,(1 + \cos\theta)$, ρ^2/r is constant. **(7)**

(c) Evaluate $\int_0^{\pi/4} \ln(1 + \tan x)\, dx$ **(6)**

Q. No.8

(a) Approximate $\int_0^2 dx/(1 + x^3)$ using trapezoidal rule with n = 4 **(6)**

(b) Find the reduction formula for $\int x^m\, (a + bx^n)^p\, dx$. **(7)**

(c) Show that the intrinsic equation of the cycloid:
$x = a\,(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$
$S = 4a \sin\alpha$. **(7)**