QUAID-I-AZAM UNIVERSITY

DEPARTMENT OF MATHEMATICS

Sample Paper: **General Mathematics** **(Paper - B)** Max Marks: 100

Note: Attempt **FIVE** questions by selecting at least ONE question from each sections.

SECTION - I

Q.1. (a) Show that the normal at any point of the curve (10)

$$x = a\cos\theta + a\theta\sin\theta,$$
$$y = a\sin - a\theta\cos\theta$$

is at a constant distance from the origin.

(b) Show that the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{1}{P} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 b^2}$. (10)

Q.2. (a) If $|\vec{a}| = |\vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$

(b) If $\vec{f} = e^z(\sin y\,\underline{i} + x\cos y\,\underline{j} + \sin y\,\underline{k})$, then verify curl $\vec{f} = 0$ and a function such that $\nabla\phi = \vec{f}$. (10)

Q.3. (a) Find equations of the perpendicular from the point $P(1,6,3)$ to the straight line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also obtain its length and coordinates of the foot of the perpendicular. (10)

(b) Find an equation of the plane containing the line $x = 2t,\ y = 3t,\ z = 4t$ and the intersection of the planes $x + y + z = 0$ and $2y - z = 0$. (10)

SECTION - II

Q.4. (a) Find the rank of the matrix (10)

$$\begin{bmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \\ 2 & 7 & -3 & 6 & 13 \end{bmatrix}$$

(b) Show that the system (10)

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= a \\ 3x_1 + x_2 - 5x_3 &= b \\ -5x_1 - 5x_2 + 21x_3 &= c \end{aligned}$$

is inconsistant if $c \neq 2a - 3b$.

Q.5. (a) Show that (10)

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

(b) Show that the yz-plane $W\{0,y,z)|y,z \in R\}$ is spanned by $(0,1,1)$ and $(0,2,-1)$. (10)

SECTION - III

Q.6. (a) Prove that $\left(\frac{1+\sin x+i\cos x}{1+\sin x-i\cos x}\right)^n = \cos n\left(\frac{\pi}{2}-x\right)+i\sin n\left(\frac{\pi}{2}-x\right)$. (10)

(b) Prove that (10)

$$\sec(x+iy) = 2\frac{\cos x\cosh y + i\sin x\ \sinh y}{\cos 2x+\cosh 2y}.$$

Q.7. (a) Solve $(2x\cos y)dx + x^2(\sec y - \sin y)dy = 0.$ (10)

(b) Prove that in a spherical triangle ABC (10)

$$\cos\frac{A}{2} = \sqrt{\frac{\sin s\sin(s-a)}{\sin b\sin c}}\ .$$