Sample Paper: **Mathematics A Course (Paper-B)** Max Marks: 100

Note: Attempt **FIVE** questions by selecting at least TWO question from each sections.

## SECTION - I (4/8)

Q.1. (a) Show that $\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent. What is its nilpotency index?

(10)

(b) If $A$ and $B$ be idepotent matrices. Show that if $AB = BA$ then $AB$ is idempotent.

Q.2. (a) For what value of $\lambda$ the equations (10)

$$(-\lambda)x_1 + x_2 - x_3 = 0$$
$$x_1 - \lambda x_2 - 2x_3 = 0$$
$$x_1 + 2x_2 - x_3 = 0$$

have non-trival solutions. Find these solutions.

(b) Solve the system of linear equatoins (10)

$$2x_1 + x_2 - 5x_3 = 4$$
$$3x_1 - 2x_2 - 2x_3 = 2$$
$$5x_1 + 8x_2 - 4x_3 = 1.$$

Q.3. (a) Without expanding show that (10)

$$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$$

(b) Let $U$ and $W$ be the subspaces of $R^3$ defined by (10)

$U = \{(x,y,z)|x = y = z\}$ and

$W = \{(0,y,z)|y,z \in R\}$.

show that $R^3 = U \oplus W$.

Q.4. (a) Find the norm of $\left[\frac{1}{2}, -\frac{1}{4}, \frac{1}{3}, \frac{1}{6}\right] \in R^4$ w.r.t. the Euclidean inner product on $R^4$. (10)

(b) If $\lambda$ is an eigenvalue of a non-singular matrix $A$, then show that $\lambda^{-1}$ is an eigen value of $\lambda^{-1}$. (10)

**SECTION - II (4/8)**

Q.5. (a) Solve $dy/dx = \frac{x+3y-5}{x-y-1}$. (10)

(b) Solve $y = Px + x^3P^2$. (10)

Q.6. (a) Solve the initial value problem (10)

$$x\ dy/dx + 3y = x^3y^2 \quad y(1) = 2.$$

(b) Find the general solutions of

$$(D^4 - 2D^3 + D)y = x^4 + 3x + 1.$$ (10)

Q.7. (a) Solve $x^2 \frac{d2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$. (10)

(b) Solve $(1 + y^2)\frac{d2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + \frac{dy}{dx} = 0$. (10)

Q.8. (a) Apply the power series method to solve the differential equation $x(1 - x)y' = y$. (10)

(b) Compute the Laplace transform of $(t^3 - 1)e^{-2t}$. (10)